Solving the Pricing Problem in Branch-and-Price using Zero-Suppressed Binary Decision Diagrams

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Pricing Problems are Problematic

- How to run branch-and-bound on IP with exponential number of variables?
- A pricing subproblem must be solved at every node in the search tree
- The pricing problem is generally incompatible with standard branching rules

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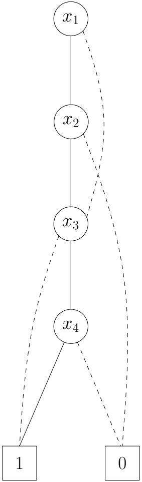
ZDDs are used to compactly encode a family of sets

- Let $\mathcal{U} = \{x_1, x_2, x_3, x_4\}$ and $\mathcal{F} = \{\varnothing, \{x_1, x_2\}, \{x_3, x_4\}, \mathcal{U}\}$
- Knuth (2008) gives an algorithm to construct the *unique* reduced ZDD for a family $\mathcal{F}$
- Hadžić and Hooker (2007) show how to solve weighted optimization problems



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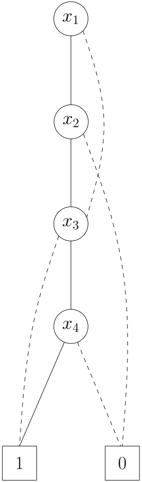

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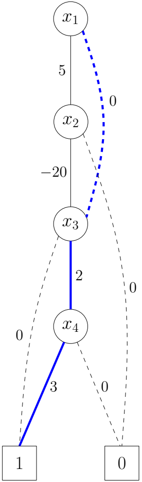

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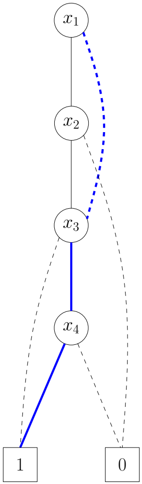
Adding Restrictions to a ZDD

Theorem

Given a ZDD Z and a set $S \in \mathcal{F}$, there is an algorithm to construct a new ZDD Z' which accepts $\mathcal{F} - S$

Runs in $O(|\mathcal{U}|)$ time, size of Z increases by at most $|\mathcal{U}|$ nodes



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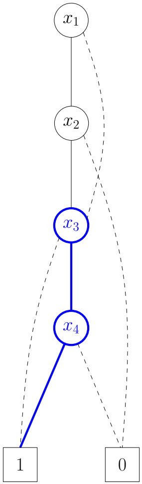

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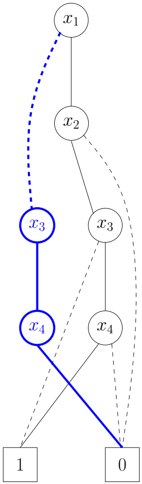

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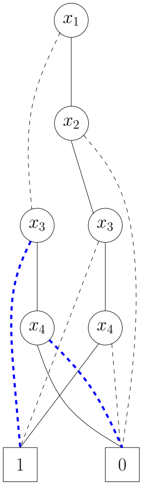

Using ZDDs to aid branch-and-price algorithms

1. Build a ZDD characterizing the pricing problem
2. Initialize the branch-and-price tree and begin the search
3. At each subproblem, use the ZDD to solve the pricing problem
4. Whenever a new column is added to the master problem, restrict it from subsequent appearance in the ZDD

What types of problems are ZDDs suited for?

Advantages of ZDDs:

- Allows standard integer branching methods to be used
- Always produces the optimal solution to the pricing problem

Disadvantages of ZDDs:

- More difficult to remove variables from column generation pool
- Sometimes ZDD is too large or too time-consuming to construct

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Computational Results on Graph Coloring

Comparison against three algorithms in the literature:

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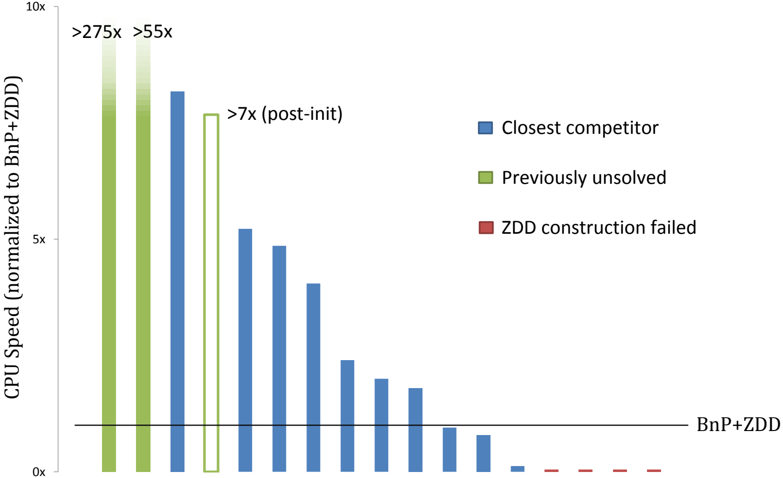

How do we handle ZDDs that are too large?

- Different orderings?
- Approximate ZDDs?
- Partial ZDDs?
- ... ?

Thank you!

Any questions?

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